量子物理学研究室

Quantum Physics Group

宮寺　隆之

量子物理学に関する　**理論**　を研究しています

Suppose that a family of observables $\{\mathsf{A}_1, \ldots, \mathsf{A}_n\}$ satisfies $\chi_{\mathrm{incomp}}(\mathsf{A}_1, \ldots, \mathsf{A}_n) = N$. Then there exists a family of linearly independent states $\{\varrho_1, \varrho_2, \ldots, \varrho_N\}$ in $\mathcal{L}_s(\mathcal{H})$ on which $\{\mathsf{A}_1, \ldots, \mathsf{A}_n\}$ are incompatible. We can regard the family $\{\mathsf{A}_1, \ldots, \mathsf{A}_n\}$ as an element of a vector space $\mathcal{L}$ defined as $\mathcal{L} := \oplus_{j=1}^n \mathcal{L}_s(\mathcal{H})^{m_j}$, that is, $\mathsf{A} := \oplus_{j=1}^n \mathsf{A}_j \in \mathcal{L}$. For each $l = 1, \ldots, N$, $j = 1, \ldots, n$, and $x_j = 1, \ldots, m_j$, let us define a subset $K(\mathsf{A}, \varrho_l, j, x_j)$ of $\mathcal{L}$ as

$$K(\mathsf{A}, \varrho_l, j, x_j) := \{\mathsf{B} \in \mathcal{L} \mid \langle \varrho_l | \mathsf{B}_j(x_j)\rangle_{HS} = \langle \varrho_l | \mathsf{A}_j(x_j)\rangle_{HS}\}, \qquad (19)$$

where $\langle \varrho_l | \mathsf{A}_j(x_j)\rangle_{HS} := \mathrm{tr}[\varrho_l \mathsf{A}_j(x_j)]$ is the Hilbert-Schmidt inner product on $\mathcal{L}_s(\mathcal{H})$. Note that this inner product can be naturally extended to an inner product $\langle\langle \cdot | \cdot \rangle\rangle$ on $\mathcal{L}$:

$$\langle\langle \mathsf{A} | \mathsf{B} \rangle\rangle = \sum_{j=1}^{n} \sum_{x_j=1}^{m_j} \langle \mathsf{A}_j(x_j) | \mathsf{B}_j(x_j)\rangle_{HS}.$$

Embedding ϱ_l into $\mathcal{L}$ by $\hat{\varrho}_l^{j,x_j} = \oplus_{i=1}^n \oplus_{y=1}^{m_i} \delta_{ij}\delta_{yx_j}\varrho_l$ for each j, x_j, and l, we obtain another representation of (19) as

$$K(\mathsf{A}, \varrho_l, j, x_j) = \{\mathsf{B} | \langle\langle \hat{\varrho}_l^{j,x} | \mathsf{B} \rangle\rangle = \langle\langle \hat{\varrho}_l^{j,x_j} | \mathsf{A} \rangle\rangle\}. \qquad (20)$$

Thus this set is a hyperplane in $\mathcal{L}$. Note that $\{\hat{\varrho}_l^{j,x}\}_{l,j,x_j}$ is a linearly independent set in $\mathcal{L}$. Consider an affine set $K := \cap_{l=1}^N \cap_{j=1}^n \cap_{x_j=1}^{m_j} K(\mathsf{A}, \varrho_l, j, x_j)$. Because $\{\mathsf{A}_1, \ldots, \mathsf{A}_n\}$ is incompatible in $\{\varrho_1, \ldots, \varrho_N\}$, it satisfies

$$K \cap C = \emptyset, \qquad (21)$$

where $C := \{\mathsf{C} \in \mathcal{L} \mid \{\mathsf{C}_1, \ldots, \mathsf{C}_n\}$ is compatible$\}$. Thus, by

with $K_0 \subset H_0$, and $\langle\langle \mathsf{C}' | h \rangle\rangle < 0$ for all $\mathsf{C}' \in C'$ the vector h. It satisfies

$$\langle\langle w_a | h \rangle\rangle = 0 \, \forall a = N\left(\sum_j m_j\right) + 1, \ldots$$

because $K_0 \subset H_0$ [see (22)]. Thus, if we $\sum_{a=1}^{\dim \mathcal{L}} c_a v_a$, then we can find that $c_a = 0$ h $N(\sum_j m_j) + 1, \ldots, \dim \mathcal{L}$. It follows that

$$h = \sum_{a=1}^{N(\sum_j m_j)} c_a v_a = \sum_l \sum_j \sum_{x_j} c_{l,j,}$$

holds and the hyperplane H_0 can be written as

$$H_0 = \left\{ \mathsf{B} \in \mathcal{L} \,\middle|\, \sum_l \sum_j \sum_{x_j} c_{l,j,x_j} \mathrm{tr}[\varrho_l \mathsf{B}_j(\right.$$

Then the hyperplane $H' := \mathsf{A} + H_0$, a translat form

$$H' = \left\{ \mathsf{B} \in \mathcal{L} \,\middle|\, \sum_l \sum_j \sum_{x_j} c_{l,j,x_j} \mathrm{tr}[\varrho_l \mathsf{B}_j(\right.$$

contains the original sets K and satisfies

$$\sum_l \sum_j \sum_{x_j} c_{l,j,x_j} \mathrm{tr}[\varrho_l \mathsf{C}_j(x_j)] <$$

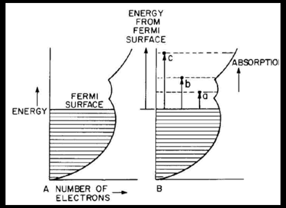
for all $\mathsf{C} \in C$. We can displace H' slightly in

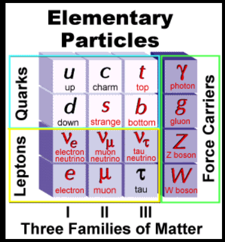
- 量子測定
- 量子情報
    - 量子暗号
    - 量子計算
- 量子アルゴリズム的情報理論
- 量子論理
- 量子カオス
- 量子非平衡状態
- 量子多体系のデコヒーレンス
- 量子論における自発的対称性の破れ
- 量子ダイナミカルエントロピー
- 量子化学における制御
- 量子細線（メゾスコピック系）
- 量子マルコフ鎖

・・・

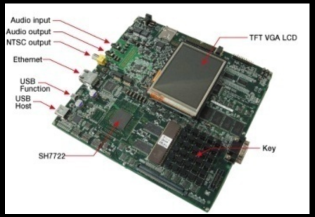
理論研究の中でも

モノの性質を調べる





モノの性質を利用する

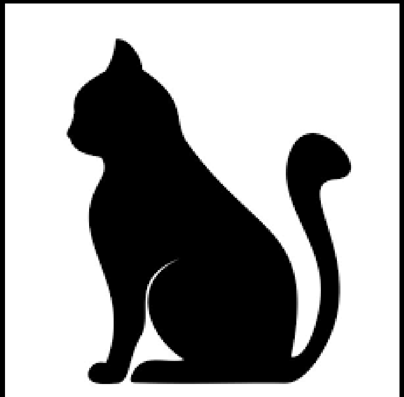


モノに依らない普遍的な性質を
調べる

モノに依らない　量子論の普遍的な特性

重ね合わせの原理

+



エンタングルメント

不確定性関係

$$\Delta q \cdot \Delta p \geq \frac{\hbar}{2}$$

状態準備の不確定性関係

$$\Delta A \cdot \Delta B \geq \frac{1}{2}|\langle [A, B] \rangle|$$

Robertson (1929)

$$H(A) + H(B) \geq -2max_{n,m} \log \|A_n^{1/2} B_m^{1/2}\|$$

Krishna-Parthasarathy (2001)

$$\sum_n \langle A_n \rangle \leq 1 + \left(\sum_{n \neq m} \|A_n^{1/2} A_m^{1/2}\|^2 \right)^{1/2}$$

Miyadera (2007)

同時測定の不確定性関係

$$2D_A D_B + D_A + D_B + 4D_A^{1/2} D_B^{1/2} \geq \max \|[A_a, B_n]\|$$

Miyadera (2008)

不確定性関係どうしの関係

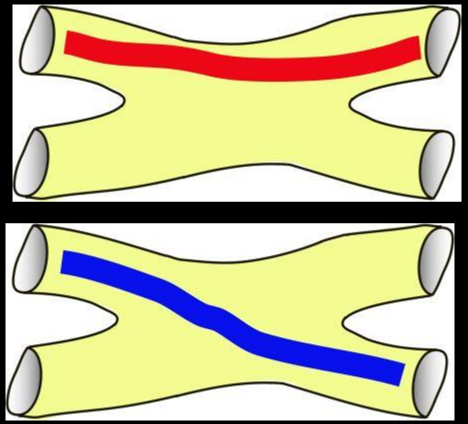
Theorem III.1. *Let Ω be a transitive state space and its positive cone V_+ be self-dual with respect to $\langle \cdot, \cdot \rangle_{GL(\Omega)}$, and let (F, G) be a pair of ideal measurements on Ω. For an arbitrary approximate joint measurement $\widetilde{M}^{FG}$ of (F, G) and $\varepsilon_1, \varepsilon_2 \in [0, 1]$ satisfying $\varepsilon_1 + \varepsilon_2 \leq 1$, there exists a state $\omega \in \Omega$ such that*

$$\mathscr{W}_{\varepsilon_1}(\widetilde{M}^F, F) \geq W_{\varepsilon_1+\varepsilon_2}(\omega^F),$$
$$\mathscr{W}_{\varepsilon_2}(\widetilde{M}^G, G) \geq W_{\varepsilon_1+\varepsilon_2}(\omega^G).$$

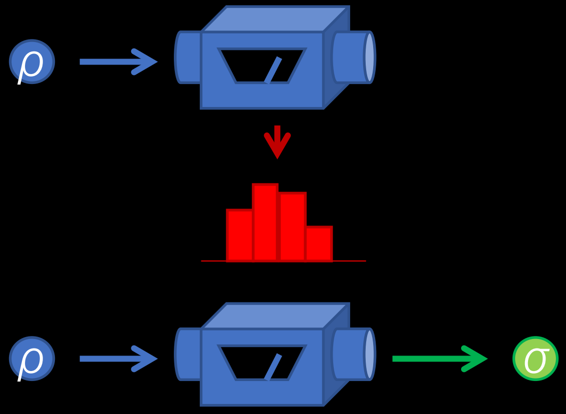
Miyadera (2011), Takakura-Miyadera (2021)

さらに一般化

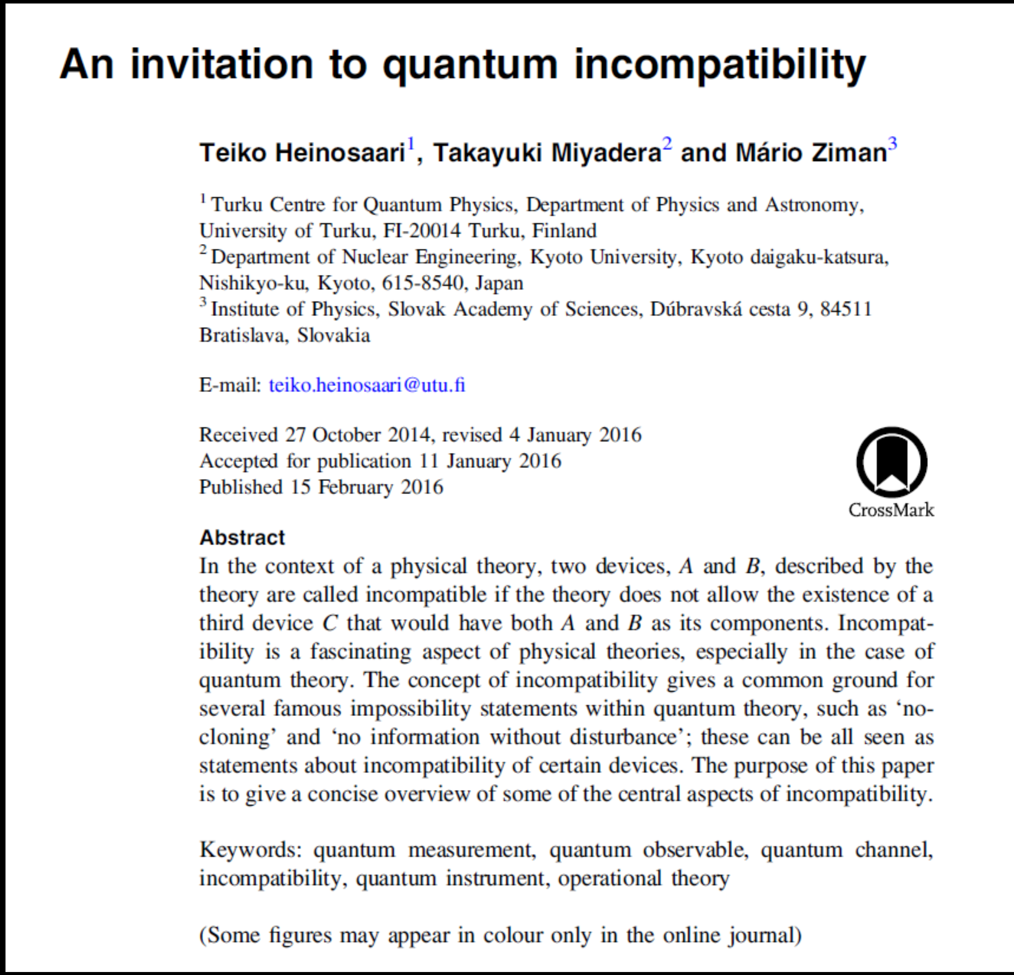
Quantum Incompatibility (2016)



Channel-channel incompatibility
(e.g., no-cloning theorem)



Information disturbance relation



An invitation to quantum incompatibility

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CrossMark

Abstract
In the context of a physical theory, two devices, *A* and *B*, described by the theory are called incompatible if the theory does not allow the existence of a third device *C* that would have both *A* and *B* as its components. Incompatibility is a fascinating aspect of physical theories, especially in the case of quantum theory. The concept of incompatibility gives a common ground for several famous impossibility statements within quantum theory, such as 'no-cloning' and 'no information without disturbance'; these can be all seen as statements about incompatibility of certain devices. The purpose of this paper is to give a concise overview of some of the central aspects of incompatibility.

Keywords: quantum measurement, quantum observable, quantum channel, incompatibility, quantum instrument, operational theory

(Some figures may appear in colour only in the online journal)

Quantum Incompatiblity

同時操作不可能性

どのような操作が同時操作可能か不可能か

同時操作不可能ならばどの程度不可能なのか

同時操作不可能性と他の量子性の関係は

同時操作不可能性は量子論を特徴づけるのか

量子論の限界を　内側から　そして外側（一般確率論）から

量子物理学研究室における最近の学生のテーマ

R.2年度

博士論文

- Conduction and diffusion of Fermi particles on lattices -from the standpoint of nonequilibrium statistical mechanics-

修士論文

- 計算後古典通信可能なマルチサーバーブラインド量子計算
- 環境支援型測定と量子測定反転によるエンタングルメントの保護および超稠密符号の改善

卒業論文

- 量子アルゴリズムによるクラス識別
- 量子状態の操作とBell nonlocality
- 時間依存した操作を含まないモデルにおける量子ゼノ効果

R.1年度

博士論文

- 量子論と一般確率論における両立不可能性と合成系の研究

修士論文

- 量子論における両立不可能性のリソースとしての性質
- 反復可能性を満たさない測定における量子ゼノ効果

卒業論文

- 量子論におけるエンタングルメントの様々な定量化
- Quantum Computation: Harmonic relation between Theoretical Computer Science and Physics
- 最大テンソル積をとる合成系での量子状態の確率的複製

量子計算（量子コンピュータ）

量子論基礎

量子測定理論

量子非平衡統計力学

.. and more

量子論を深く学びたい人

基礎が好きな人

基礎がわからないと納得いかない人

理論が好きな人

数学が好きな人

原子核コース
量子物理学研究室　へ

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